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FINAL REPORT
High-Order Algorithms for 3D Plasma Simulations
on Unstructured and Hybrid Meshes
AFOSR number: F49620-97-1-0185

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Objectives

- Develop high-order discretizations on unstructured and hybrid meshes for the viscous unsteady MHD equations in two- and three-dimensions. No such capability exists in the AFOSR labs or elsewhere in the world.
- Advance the state-of-the-art in multi-resolution algorithms and parallel software.
- Apply the new capability to parallel simulations of isothermal and MHD turbulence in complex-geometry domains.

Accomplishments/New Findings

The work completed under this project builds on our previous work sponsored by AFOSR on developing a *universal* spectral basis for formulating algorithms for high-order accurate solutions of arbitrary nonlinear systems of PDEs in complex-geometry domains. *To this end, several classes of classical numerical methods (e.g. finite elements and finite volumes) as well as modern methods (e.g. spectral and spectral elements) are simply subcases of the developed new numerical methodology.* One of the unique features of the new method is that its accuracy is insensitive to the mesh quality, which allows simulation in moving and very distorted domains (without re-gridding) at extremely high-efficiencies, not possible with any other method.

The specific tasks completed include:

- Development of Discontinuous Galerkin projections. This is a *breakthrough* in variational numerical methods, which will have great impact in many AFOSR applications. It allows general high-order differential operators to be discretized with finite jumps (L^2 continuity) across subdomains. This, in turn, implies that "sliding" domains (of great interest to weapons simulation technology) can now be discretized with no extra computational effort. In addition, intelligent trial bases can be used in different subdomains to deal with singularities, discontinuities and sharp gradients while optimizing their efficiency locally without global inter-dependence. The reduced continuity requirements make the method *inherently parallel* and naturally suited to modern parallel platforms at AFOSR labs.
- Development of Discontinuous Galerkin methods for the viscous MHD equations for compressible and incompressible turbulence, and demonstration of the high-order accuracy of the method. The parallel code we developed is the only MHD code with high-order capability in complex-geometry domains. The new formulation is extension

to high-order and unsteady flows of the work of Peterkin et al. (Kirkland AFB), Shumlak (University of Washington) and Powell et al. (University of Michigan), who are the others PIs in the AFOSR Computational Math program. In fact, their algorithms are a subset of the new formulation in the degenerate case of setting the spectral order equal to zero, which is the finite volume limit of the discontinuous Galerkin formulation. Therefore, this makes for an easy porting of our numerical algorithms to the current MACH2 and MACH3 (AFOSR codes), which are only limited to second-order accuracy at best and only cartesian grids. Our algorithm is suitable for cartesian grids, unstructured grids, as well as hybrid grids consisting of polymorphic domains.

Personnel

- Faculty: G.E. Karniadakis (US), Professor of Applied Mathematics
- PhD Students: T.C. Warburton and I. Lomtev

Publications

1. T.C. Warburton, "Spectral/hp Methods on Polymorphic Multi-Domains: Algorithms and Applications", PhD thesis, Brown University, 1998 (supervised by the PI).
2. T.C. Warburton and G.E. Karniadakis, "A discontinuous Galerkin method for the viscous MHD equations", J. Comp. Phys., in press, 1999.
3. T.C. Warburton, S.J. Sherwin and G.E. Karniadakis, "Spectral basis functions for 2D hybrid hp elements", SIAM J. Scientific Computing, in press.
4. S.J. Spencer, T.C. Warburton, and G.E. Karniadakis, "Spectral/hp methods for elliptic problems on hybrid grids", Contemporary Mathematics, vol. 218, p. 191-215, 1998.
5. T.C. Warburton, I. Lomtev, M. Kirby, and G.E. Karniadakis, "A discontinuous Galerkin method for the compressible Navier-Stokes equations on hybrid grids", Proc. Tenth International Conference on Finite Elements in Fluids, January 5-8, 1998, Tucson, Arizona, p. 604, Eds. M. Hafez and J.C. Heinrich.

